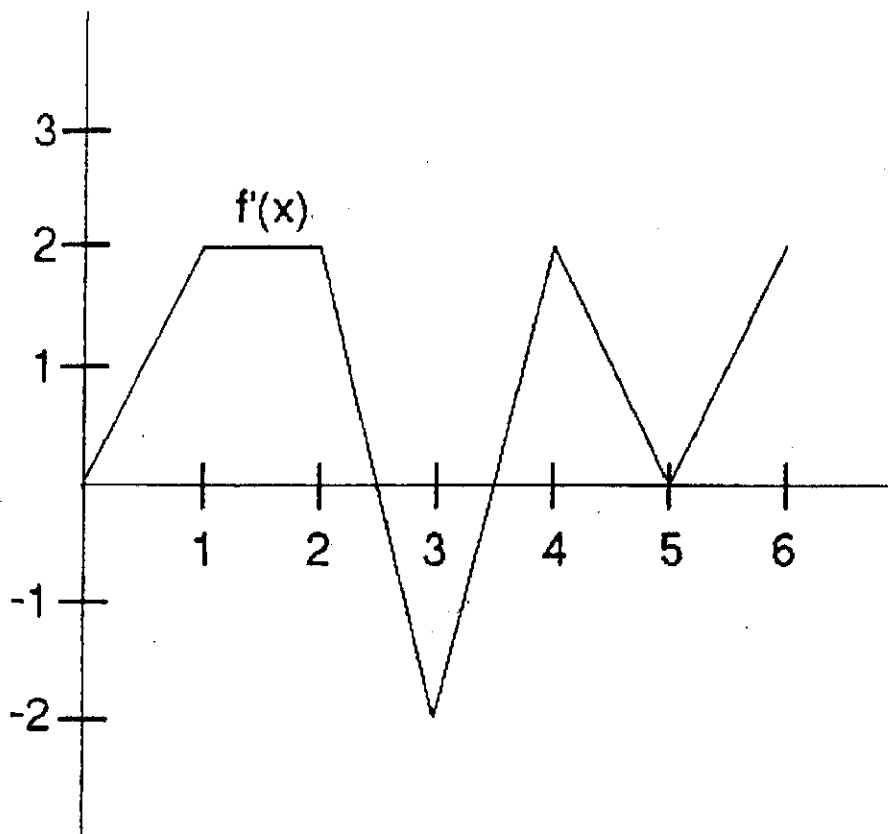


## From $f'$ to $f$



In this activity you will play detective, deducing information about a function  $f(x)$ , the graph of whose derivative  $f'(x)$  is depicted above.

- (1)(a) On which intervals is  $f(x)$  increasing?  
(b) How do you know?

- (c) On which intervals is  $f(x)$  decreasing?  
(d) How do you know?

- (2)(a) On which intervals is  $f(x)$  concave up?  
(b) How do you know?

- (3)(a) Where does  $f(x)$  have a local max?  
(b) How do you know?

- (c) On which intervals is  $f(x)$  concave down?  
(d) How do you know?

- (c) Where does  $f(x)$  have a local min?  
(d) How do you know?

- (e) List the inflection points of  $f(x)$   
(f) Why are these inflection points?

- (4) Suppose we also know that  $f(0)=0$ . Give a sketch the graph of  $f(x)$  in the area above.

In problems 1-6, assume that  $f$  is differentiable for all  $x$ . The signs of  $f'$  are as follows.

$$f'(x) > 0 \text{ on } (-\infty, -4)$$

$$f'(x) < 0 \text{ on } (-4, 6)$$

$$f'(x) > 0 \text{ on } (6, \infty)$$

Apply the appropriate inequality sign for the indicated value of  $c$ .

<u>Function</u>	<u>sign of <math>g'(c)</math></u>
1. $g(x) = f(x) + 5$	$g'(0)$ ____ 0
2. $g(x) = 3f(x) - 3$	$g'(-5)$ ____ 0
3. $g(x) = -f(x)$	$g'(-6)$ ____ 0
4. $g(x) = -f(x)$	$g'(0)$ ____ 0
5. $g'(x) = f(x - 10)$	$g'(0)$ ____ 0
6. $g'(x) = f(x - 10)$	$g'(8)$ ____ 0

7. Sketch the graph of an arbitrary function  $f$  such that:

$$f'(x) \begin{cases} > 0 & x < 4 \\ \text{undefined} & x = 4 \\ < 0 & x > 4 \end{cases}$$

8. A differentiable function  $f$  has one critical number at  $x = 5$ . Identify the relative extrema of  $f$  at the critical number if  $f'(4) = -2.5$  and  $f'(6) = 3$
9. Consider a function  $f$  such that  $f'$  is increasing. Sketch the graphs for  $f$  for (a)  $f' < 0$  and (b)  $f' > 0$
10. Consider a function  $f$  such that  $f'$  is decreasing. Sketch the graphs for  $f$  for (a)  $f' < 0$  and (b)  $f' > 0$
11.  $S$  represents weekly sales of a product. What can be said of  $S'$  and  $S''$  for each of the following statements?
- The rate of change of sales is increasing.
  - Sales are increasing at a slower rate.
  - Sales are steady
  - Sales are declining, but at a slower rate.
  - Sales have bottomed out and have started to rise.